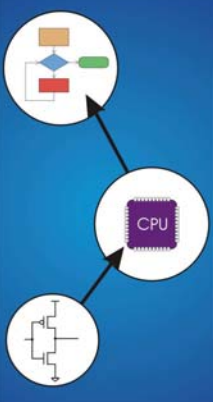




# Introduction to Computer Engineering

ECE/CS 252, Fall 2010  
 Prof. Mikko Lipasti  
 Department of Electrical and Computer Engineering  
 University of Wisconsin – Madison



## Chapter 3

## Digital Logic Structures

### - Part 2

Slides based on set prepared by  
 Gregory T. Byrd, North Carolina State University

### Building Functions from Logic Gates

We've already seen how to implement truth tables using AND, OR, and NOT, etc. -- examples of **combinational logic**.

#### Combinational Logic Circuit

- output depends only on the current inputs
- stateless

#### Sequential Logic Circuit

- output depends on the sequence of inputs (past and present)
- stores information (state) from past inputs

Next we'll show how to use sequential circuits to store information.

### Combinational vs. Sequential

#### Combinational Circuit

- always gives the same output for a given set of inputs
  - ex: adder always generates sum and carry, regardless of previous inputs

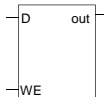
#### Sequential Circuit

- stores information
- output depends on stored information (state) plus input
  - so a given input might produce different outputs, depending on the stored information
- *example:* ticket counter
  - advances when you push the button
  - output depends on previous state
- useful for building "memory" elements and "state machines"

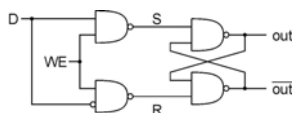
### Gated D-Latch

Two inputs: D (data) and WE (write enable)

- when **WE = 1**, latch is set to **value of D**
- when **WE = 0**, latch holds **previous value**



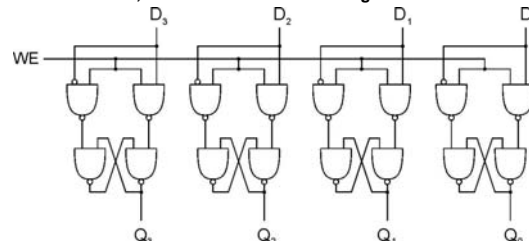
Watch online lecture to understand internals:



### Register

A register stores a multi-bit value.

- We use a collection of D-latches, all controlled by a common WE.
- When WE=1, n-bit value D is written to register.



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## Memory: 2-dimensional register

We can build a memory – a logical  $k \times m$  array of stored bits.

**Address Space:**  
number of locations  
(usually a power of 2)

$k = 2^n$  locations

**Addressability:**  
number of bits per location  
(e.g., byte-addressable)

$m$  bits

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## State Machine

Important type of sequential circuit

- Combines combinational logic with storage
- “Remembers” state, and changes output (and state) based on **inputs** and **current state**

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## Combinational vs. Sequential

Two types of “combination” locks

**Combinational**  
Success depends only on the **values**, not the order in which they are set.

**Sequential**  
Success depends on the **sequence** of values (e.g., R-13, L-22, R-3).

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## State

The **state** of a system is a **snapshot** of **all the relevant elements** of the system at the moment the snapshot is taken.

**Examples:**

- The state of a basketball game can be represented by the scoreboard.
  - Number of points, time remaining, possession, etc.
- The state of a tic-tac-toe game can be represented by the placement of X's and O's on the board.

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## State of Sequential Lock

Our lock example has four different states, labelled A-D:

**A:** The lock is **not open**, and no relevant operations have been performed.

**B:** The lock is **not open**, and the user has completed the **R-13** operation.

**C:** The lock is **not open**, and the user has completed **R-13**, followed by **L-22**.

**D:** The lock is **open**.

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## State Diagram

Shows **states** and **actions** that cause a **transition** between states.

## Finite State Machine

A description of a system with the following components:

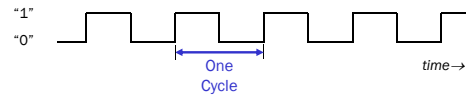
1. A finite number of **states**
2. A finite number of external **inputs**
3. A finite number of external **outputs**
4. An explicit specification of all **state transitions**
5. An explicit specification of what causes each external **output value**.

Often described by a state diagram.

- Inputs may cause state transitions.
- Outputs are associated with each state (or with each transition).

## The Clock

Frequently, a **clock circuit** triggers transition from one state to the next.



At the beginning of each clock cycle, state machine makes a transition, based on the current state and the external inputs.

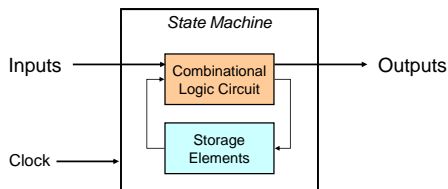
## Implementing a Finite State Machine

### Combinational logic

- Determine outputs and next state.

### Storage elements

- Maintain state representation.



## Storage

Each storage element remembers one state bit.

The number of storage elements needed is determined by the number of states (and the representation of each state).

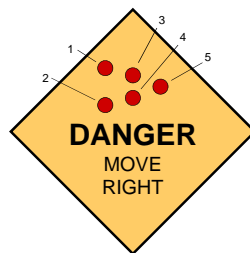
### Examples:

- Sequential lock
  - Four states – two bits
- Basketball scoreboard
  - 7 bits for each score, 5 bits for minutes, 6 bits for seconds, 1 bit for possession arrow, 1 bit for half, ...

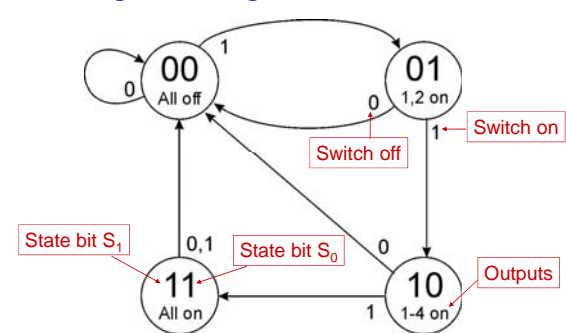
## Complete Example

### A blinking traffic sign

- No lights on
- 1 & 2 on
- 1, 2, 3, & 4 on
- 1, 2, 3, 4, & 5 on
- (repeat as long as switch is turned on)



## Traffic Sign State Diagram



Transition on each clock cycle.

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### Traffic Sign Truth Tables

Outputs  
(depend only on state:  $S_1S_0$ )

Next State:  $S_1'S_0'$   
(depend on state and input)

Switch

| $S_1$ | $S_0$ | Z | Y | X |
|-------|-------|---|---|---|
| 0     | 0     | 0 | 0 | 0 |
| 0     | 1     | 1 | 0 | 0 |
| 1     | 0     | 1 | 1 | 0 |
| 1     | 1     | 1 | 1 | 1 |

Lights 1 and 2  
Lights 3 and 4  
Light 5

| In | $S_1$ | $S_0$ | $S_1'$ | $S_0'$ |
|----|-------|-------|--------|--------|
| 0  | X     | X     | 0      | 0      |
| 1  | 0     | 0     | 0      | 1      |
| 1  | 0     | 1     | 1      | 0      |
| 1  | 1     | 0     | 1      | 1      |
| 1  | 1     | 1     | 0      | 0      |

Whenever  $In=0$ , next state is 00.

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### Traffic Sign Logic

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### From Logic to Data Path

The data path of a computer is all the logic used to process information.

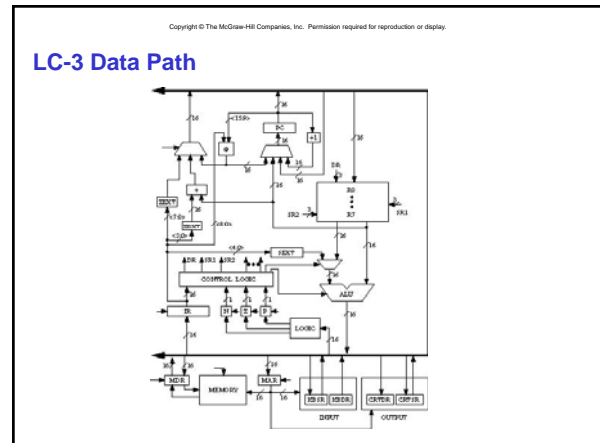
- See the data path of the LC-3 on next slide.

#### Combinational Logic

- Decoders -- convert instructions into control signals
- Multiplexers -- select inputs and outputs
- ALU (Arithmetic and Logic Unit) -- operations on data

#### Sequential Logic

- State machine -- coordinate control signals and data movement
- Registers and latches -- storage elements



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### Summary

#### Sequential Logic Circuits

##### Storage/Memory

- D Latch
- Register
- Memory
- Watch online lecture for more details

##### Finite State Machines

- State Diagram
- Output Logic
- Next State Logic

##### LC-3 Datapath